INSTRUCTIONAL DESIGN OF COLLEGE ALGEBRA MODULE

Methods for Solving Quadradic Equations

The goal of this piece of instruction is to design a learning module for a college algebra course. We will be focusing specifically on the topic of solving quadratic equations. This topic includes various methods that are important for college learners as they can be both applied to real-world situations and provide a solid foundation of understanding that is key for future concepts. It is aimed at an entry-level college student who are either in their first years as a mathematics or science major as well as non-natural science majors in need of a mathematics requirement. The following instructional design plan provides a clear outline for implementation of said module. Upon student completion of the module, learners will be able to effectively solve any type of quadratic equation as well as real world applications of the topic. In addition, learners will feel confident in their understanding of quadratic equations and able to apply knowledge gained outside the course.

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PART 1: ANALYSIS

Needs Assessment

While the topic of solving quadratic equations falls under a basic instructional need for early college learners, I too believe that it represents a basic problem in college algebra. This is because it normally encompasses one or two modules, within fourteen to sixteen for the course as a whole. It is presented as simply another topic, but in truth the ability to understand and effectively solve equations of this nature is a basic building block for both the remainder of the course as well as future mathematics courses and real-world problems. In this design, my goal is to enrich this module in order to ensure success for learners both in the college algebra course and beyond. Robert Blumenthal (2016), states, “for many schools, this [college algebra] is the math requirement.” While he goes on to question if this is the right choice, and suggests alternate courses such as Quantitative Reasoning, to replace this requirement, the fact remains that most students are required to pass this course to obtain a degree. Moreso, he states that the topics covered in college algebra are, “essential for students who will progress to calculus.” In addition, a student replying to the article states, “many of [the topics] are crucial for technology, physics etc.” While we could continue to elaborate on the need of solid instruction of college algebra in universities, instead we will accept the premise that based on the prevalence of college algebra as a requirement, it represents an instructional need. Further, based on my own knowledge of mathematics (I hold two degrees in the subject and am currently working toward a graduate certificate), I have identified this module as a key topic that needs to be developed.

For this analysis, we will be using Smith and Ragan’s Discrepancy-Based Needs Assessment Model. We will complete the following five steps as outlined by Brown and Green (2016, p.53).

1. List the goals of the instructional system.
2. Determine how well the identified goals are already being achieved.
3. Determine gaps between “what is” and “what should be.”
4. Prioritize gaps according to agreed-upon criteria.
5. Determine which gaps are instructional needs and which are most appropriate for design and development of instruction.

In order to work through the above steps, we will need to collect actual data from students currently enrolled in a college algebra course. We will begin by taking the learning goals already established for this course that relate to our specific module. This will include both the goals of the module we seek to improve as well as learning objectives from later modules that rely on mastery of the topic. In order to determine how well these goals are being achieved we will examine homework and quiz scores for the module. We will also conduct a mathematical analysis comparing these scores to the final course grade to determine if there is any clear correlation between lack of understanding of quadratic equations and overall poor performance in the course. In attempting to determine gaps, we will devise an acceptable level of understanding of quadratic equations. For instance, completion of homework and quizzes for the given module with a score of 85% or higher. We will then examine all students who did not

Entry Behaviors

As this is a module for an entry-level college course, our students will maintain a high school diploma or equivalent. Due to the varying nature of mathematics completion in secondary school, a placement exam is required for enrollment in the course. Thus, students must have a basic knowledge of the concepts covered in Algebra 1 and 2 courses from high school including:

- Linear Equations
- Exponents
- Functions
- Factoring

Goal Statement:

Upon completion of the module, students will have a solid foundational understanding of quadratic equations including, but not limited to, the ability to use various methods to solve.

In addition, as this course is offered in an online format, students must have a basic understanding of computer applications, internet, email and word processing. While it may be difficult for some students as discussed below, learners also need to enter the course with a readiness and willingness to learn. As well as the ability to devote three to five hours per week on the course (our particular module will consist of two weeks). Also, due to the nature of the instruction, students must be self-motivated to participate in discussions, office hours, complete homework assignments and watch lecture videos independently.

The Learners

As Brown and Green state (2016), "In order to gain understanding of the target audience, one must conduct some form of preliminary evaluation of that group (p. 73)." As I have not had the opportunity to gather actual information form my target group, the topics discussed here will be mainly based on assumption. However, in implementation of this design plan, my goal would be to conduct a pre-survey to have accurate information in order to augment some areas appropriately.

Learner Attitudes / Motivation

Most students taking College Algebra do so for one of the following reasons:

1. The student is interested in mathematics and has elective credits to fulfil. They enter the course with a positive attitude, excited to learn more about the concepts. **(Willing Volunteer / Intrinsic Motivation)**
2. The student is interested in or pursuing a mathematics degree and the course is a requirement in order to enroll in other upper-level courses. These students also have a love of mathematics and are very motivated to create a solid foundation on which to build subsequent knowledge. We classify this learner as captive because the course is a requirement and may not be of particular interest to the student, however as the topic is mathematics, a subject in which they have a high level of interest, we describe their motivation as intrinsic. **(Captive Audience/ Intrinsic Motivation)**

differences in motivation, learners will fall into one of the categories providing similarities within said group. Lastly, while there will be differences in learning styles, all students will share the preference to receive instruction online. Reasonings for this preference will vary (location, time, etc.), but students will all be prepared for the format of instruction.

Context and Implications for Instruction

The instruction for this course/module will take place in the Canvas learning platform. It will be entirely online. There will be no physical element to the learning site. While the majority of instruction will be independent, students will be required to participate in class discussions incorporating a social element to the module as well as adding some of the benefit of in-class learning through collaboration. Thus, there will be student-student interaction. Learning will be facilitated via instructional videos and there will be due dates for each portion. In addition, there will be two to four office hour sessions during the module. This will provide students with the opportunity to speak directly to the instructor in e-classroom environment. These will take place in Adobe Connect. Lastly, we will be utilizing the Pearson website *MyMathLab* for homework assignments. This resource includes an interactive textbook as well as practice problems for students to use as needed.

Part 2: Objectives

Instructional Goal

The instructional goal of our design is to improve the existing college algebra module on quadratic equations. By the end of the module students will not only be able to effectively solve quadratic equations, but also apply their knowledge to real-world problems. Students will show a mastery of the topic which represents comprehension rather than the ability to mimic steps to arrive at a correct solution.

Learning Objectives

QE-1. Students will have the ability to locate or identify quadratic equations.

We can also state this as, given a list of equations, students will determine which are quadratic. We have identified this as our first key objective because before students can move onto solving and understanding quadratic equations, they first must be able to identify the components of said equation. In order to do this, students must have a basic understanding of mathematical format, equations and exponents.

QE-1.2. Students will restate given quadratic equations in general form.

While the ability to rewrite a quadratic equation falls low on the scale of importance to comprehension of the topic, we include it here because it can help some students in

Also, introductory skills of algebra are needed as we must apply to both sides of the equation to maintain equality.

QE-2.3. Given any quadratic equation, students will solve using the quadratic formula.

This method is key as we can solve any quadratic equation using the provided formula. Due to its importance, this objective should be implemented in its own mini-series of Bloom's revised taxonomy as follows:

- **Remember:** Students will memorize the quadratic formula.
- **Understand:** Students will explain how the quadratic formula is derived using the complete the square method.
- **Apply:** Given quadratic equations, students will solve using the quadratic formula.
- **Analyze:** Students will compare and contrast the quadratic formula with other methods of solving.
- **Evaluate:** Students will weigh the lengthy process of the quadratic formula with other methods of solution to determine when the quadratic formula is best applied.

In order to master, students must enter with knowledge of using and applying a formula as well as a deep understanding of quadratic equations from previous objectives.

QE-2.3.1. Given a quadratic equation, students will identify the discriminant and explain how its value determines the number and type of solutions.

This objective helps students to classify their answers, thus we remain in the *apply* stage. It also marks the beginning of a transfer to *analyzation*, as they acquire the ability to identify different types of solutions, and the *evaluate* stage as they begin to asses if their solution falls within the correct parameters. Students must have an understanding of inequalities and complex numbers to excel in this objective.

QE-3. Given a quadratic equation, students will identify the best method of solution.

While this objective is somewhat flexible as students can choose their method based on their own learning style and what works best for them, it is also important to point out the benefits and limitations of each method to better inform their choice. While we began this process in QE-2.3, now as the students have mastered all forms of solution, they are better equipped to *analyze* using the compare and contrast method. They will also graduate to the *evaluate* stage as they decide which method to use. Students will need basic problem-solving skills as well as a mastery of previous objectives. We note that this objective is key to the process and a good amount of time should be devoted to mastery. Below are some possible activities to include when completing this objective.

- Students can complete a graphic using technology to create a chart or infographic on each method and the pros and cons.
- Provide students with many different examples of quadratic equations in which they justify the method they would use.

Manipulatives

Why do we introduce students to different methods of solution of quadradic equations? There are many different answers and opinions on this question. We will address those that drive this piece of instruction.

1. *Provide opportunities for different learning styles.*
2. *Introduce students to the manipulative nature of mathematics.*
3. *Incorporate prerequisite skills while providing the student with the knowledge needed for future mathematical concepts.*

Some students will struggle with a particular method. By providing multiple ways to solve, students can hone in on what works best for them, allowing for better understanding. Unfortunately, while many of the equations we tackle in college algebra can be solved by factoring, some are almost impossible to approach this way. Thus, while students who enter the course with a high understanding of multiples and factors will prefer this method, it is important we provide additional tools as factoring is not a universal key. Sönnerhed (2011) writes, "Some mathematics education articles advise mathematics teachers to simplify ancient-time mathematician al-Khwarizmi's method of completing the square based on the geometrical ideas and then present the simplified version to students (p.31)." It is important to note that the study of algebra is a long one. Incorporating a small portion of its history may help students to connect its importance to the outside world. In fact, when first developed, algebra was bred out of necessity to calculate everyday activities. Thus, as Sönnerhed (2011) states, introducing students to these ancient methods may very well help to both connect the concepts to real-world scenarios, and also provide a different representation for multiple types of learners. It was not until the 17th century when Descartes connected geometric understanding to mathematical equations as we see in today's algebra. This gives students the opportunity to visualize shapes and diagrams into equations, which can be a very powerful tool.

While each method comes with its own set of prerequisite skills, instructing students in all methods ensures they connect previous topics. This is key as students graduate from quadratic equations onto other topics they will need to be comfortable in these objectives to excel. Providing a continuing theme of building concepts helps to guide students to automatically consider what they have already learned in order to connect it all together.

Lastly, our goal is to not just teach students "how to" solve, but instill an understanding of what a quadradic equation is, why each step is performed, and how we can apply this knowledge. Sönnerhed (2011) writes, "In the USA, the standard quadratic formula is regarded as the standard method for solving quadratic equations (p.37)." It is true that the quadratic formula is a valuable tool as it can be applied to any quadradic function whereas other methods are only valuable for equations in a certain form. However, only focusing on this method is a disservice to students. It would require little graduation in the hierarchy of learning as presented by Bloom as after memorization of the method, students would not have the opportunity to analyze for a deeper understanding of quadratic equations as a whole. In addition, it leaves out many possibilities for different types of learners to "play" with these equations in order to familiarize themselves with their properties. Sönnerhed (2011) accurately describes, "The standard method might be regarded as an efficient and direct method, but it may lead students to solve quadratic equations in a mechanical way (p.37)." This "mechanical" method of mathematics focuses on applying given steps rather than comprehension of a topic.

For these reasons, we include four methods of solutions to provide to our learners. However, we encourage them to investigate each method for themselves and determine which works best for them and their style of learning.

Part 3: Instructional Strategies

Strategies and Activities for Objectives

Given the asynchronous online platform, our strategies for mastery of the objectives will be the same or very similar across the board. The strategies we will impose are summarized in the table below followed by justification for our choices.

Learning Block	Strategies
Block 1: QE-1 through QE-1.3	• Lecture Video • *MyMathLab* Homework
Block 2: QE-2 through QE-2.2	• Lecture Video • *MyMathLab* Homework • Discussion • Online Office Hour
Block 3: QE-2.3 through QE-2.3.1	• Lecture Video • *MyMathLab* Homework • Group Project
Block 4: QE-3 through QE-4	• Micro Lecture • Presentation Project • Real-World Scenario • Discussion / Feedback Forum

Block 1: QE-1 through QE-1.3

Here our goal is to have students recognize what a quadratic equation is, write it in general form and define it using proper mathematics vocabulary. Many students entering the course will already be familiar with these objectives from high school algebra. In addition, as our learners are college students, we do not wish to spend too much time on this topic in order to break it down into an elementary fashion. Instead, we wish to review and deepen knowledge. Hence, our strategies for this block will be a lecture video in which students can download at any time to watch and take notes as well as refer to when needed. We will also employ the website *MyMathLab* from Person to provide homework activities that allow the student to practice this knowledge. As we believe students will enter the course having gone through the *Remember* and *Understand* phase of Bloom's revised taxonomy previously, our focus will be activities that aid in the remaining phases of understanding. For instance, the *MyMathLab* problems will have students examine given equations, classify into quadratic and non-quadratic, identify the key components of a quadratic equation, etc.

group project as well as when working on their justifications and presentations in the discussion forum.

Instructional Media

This course will rely heavily on media. We will use instructional videos as well as infographics to present the concepts to students. Students will also have the ability to choose the type of media presentation the prefer when completing their individual presentation.

Assessment Strategy

As described above, assessment will come in a multitude of ways. Their homework score will be determined by *MyMathLab*. It is recommended that the instructor place a minimum requirement such as 80%. This will help to motivate students to rework any problems with errors to achieve a higher level of understanding. The instructor will also asses the discussion forums. While points will be given for participation, upon reading the posts and response the instructor will have a good idea of each student's level of understanding and can offer intervention where needed. The group project will be assed by the given rubric and the presentation will follow a similar guideline. By utilizing various forms of instruction, it should be clear if the student has mastered each objective. Should a student show signs of under achievement, it is recommended the instructor intervene by either offering one-on-one assistance, or referring the student to tutoring services etc. The pairing for the group project should also be intentional, ranking students based on performance in the previous objectives and creating mixed groups where more advanced students have the opportunity to guide those in need of assistance.

Part 4: Implementation & Evaluation Plan

Implementation Plan

Implementation of our design plan will be fairly straightforward and easy for any instructor to adopt. We have designed this piece of instruction to be included in an online course, thus a primary learning platform is needed, such as Canvas or Blackboard, for students to participate in discussions and connect for group projects as well as a space to submit their individual project. In addition, all students will need to purchase a Pearson account in order to complete the homework through *MyMathLab*. We recommend that the lecture videos associated with this module be available at any time. In the desired learning platform, instructors should have a tab for media. Video lectures for each objective should be included here and clearly labeled. For greatest student results, the videos should be assigned for student viewing early in the module. They should remain open in order for students to re-assess as necessary. This supports the learning goal as it allows participants to first get an understanding of the concept through instruction, then when completing homework or other course assignments, they can re-watch to increase mastery and clear up any questions that may arise. Students should only attempt the homework after watching the lecture video. By encouraging students to complete in this order, we are allowing for full development of each stage of Bloom's revised taxonomy. Should they first attempt the homework, they will not yet have mastered the primary stage; to be able to remember, describe and name the associated topics. If possible, including a tracker where the learning platform takes a virtual attendance when the lecture is accessed and keeping the homework link locked until this is done is one possible way to ensure students begin on the right track.

- Split the course into groups. It is natural that these be the same groups as created for discussion forums. Then, certain questions will have already been addressed in discussion allowing for more time in office hours for other concerns or topics that students were not able to come to a consensus on in their discussion.
- Offer different time blocks for each group. This allows for a more intimate experience where students can actively interact with the professor or teaching assistant.
- Encourage students to come prepared with questions as to not waste time.
- Post and/or record information in all discussion forums relating to what was addressed. This way, if for instance office hours are divided such that Group 1 meets Wednesdays and Group 2 Fridays, questions answered in Group 1's time slot will not need to be readdressed in Group 2's time as the recorded session is available to aid in Group 2's knowledge.

We also encourage office hours to be made a requirement, meaning attendance points are awarded that contributes to the students' overall grade.

Formative Evaluation

As a traditional college course, we will use testing to evaluate learner achievement. We recommend the use of evaluation both during the instructional process and afterward. While Brown and Green (2016) suggest that post-instruction evaluation is summative, we will use summative evaluation to determine the effectiveness of the model as a whole with formative evaluation being directed at learner achievement. This is because Brown and Green (2016) state, "Formative evaluation is used throughout the instructional design process to gather data that can be used to provide feedback on how the process is going." As our goal is to increase learner comprehension of solving quadratic equations, this can be best measure through determination of student success in the module. Because of the short duration, formal testing or quizzes is not recommended as it takes time away from the already described activities. Instead, to conduct this evaluation we suggest instructor participation and intervention in the activities. This can be done through the grading in *MyMathLab*. As previously stated, requiring a passing grade of 75% will ensure that students are gaining meaningful instruction and comprehension. Should they struggle to achieve this, the instructor should provide feedback, recommend tutoring or have the teaching assistant reach out to help individual students succeed. Should there be a trend of multiple students failing to achieve this goal, it is important to revisit the created lecture videos to ensure that they contain all necessary information, examples and that they are clear and easy to understand. Instructor participation in the discussion forums will also give a good idea of what topics are not being completely understood and where improvements can be made. Feedback on the projects will both give a chance for intervention to ensure the success of the module as well as another opportunity to find any additional gaps in understanding that need to be corrected. All of the above mentioned opportunities will impact the design plan as they will help us to identify if the instruction is effective and provide the chance to augment the lecture videos to ensure that they are of the top quality for student achievement. We intend this change of module to act as a pilot prior to assimilation in all college algebra courses. Our goal is to make the above changes in one section of the course at one university. After completing evaluation and interpreting the data, we would make necessary changes. The course would remain the same except for problems identified and again reevaluated. If at this time it is deemed successful, we would expand the scope to all college algebra sections at said university. We would also hope to collaborate with other universities and share our successes for implementation there as well. Should we find that we are unable to significantly raise understanding, we would need to reevaluate and determine if it is fiscally responsible to continue to develop this module and improve on it or if a new design plan is needed.

Summative Evaluation

We will use Kirkpatrick's model for our summative evaluation. We choose this because it is straightforward and believe it will be the most universal and easily applied process. First, we will measure reaction, or how students would rate the module. The best way to achieve this is with an exit survey. Students will be asked about this module as well as others and given the chance to compare them. Receiving positive comments about retention, understanding and ease of comprehension of the concepts being presented will help us to determine that our plan is successful. Should this not be the case, we must return to the analysis phase to identify problems with the design and rework them for optimal success. The next step is to evaluate learning. This can be done through careful examination of the final exam. Should we see that students did better overall on questions involving the quadratic method and solving such equations, we will know that our design was effective. Next, we will observe behavior. This may be difficult for this piece of instruction. However, observing how students do in subsequent modules that rely on their knowledge gained by comprehension of solving quadratic equations will be a good gauge. In the long term, also conducting studies on how students do in mathematics courses taken after college algebra that are connected to their topics and comparing to student achievement who did not participate in this updated module is a great way to determine its success. Lastly, we will look to results. Overall achievement in the course, in the particular module, and again long term in related courses will be the standard for this portion of evaluation. Overall, the above evaluations will determine if our updated design module be applied to all sections of college algebra as well as define areas that need to be changed to meet our stated goal. Most of this change should start at the lecture video level, we believe that should these videos be created in such a way that they are complete and through it will ensure success throughout the remaining activities.

Part 5: Reflection

I greatly enjoyed the process of creating this piece of instructional design. It was much harder and in-depth than I initially believed. However, pulling from the course resources created a good guide of what to think about during the process. I do wish that we were provided with a sample instructional design plan that has been completed. While we had bits and pieces I found it hard to compile all the necessary information as well as know when I had considered all parts to the fullest extent. It seems like there is always the possibility of digging deeper, finding more research on the topic, comparing and contrasting ideas, etc. Also, I struggled with the hypothetical nature of many of the concepts. As a mathematician, I am used to tangible data. This project involved a lot of, "if this happens then I will do this" but I feel as though there are an infinite number of these conditional statements. I would feel much more comfortable with a bit more direction and possibly even peer review early in the process. However, overall having the chance to actually implement many of the concepts we learned over the semester was greatly beneficial. It took topics out of the extensional and into something tangible, which I appreciate. I still feel unsure if I have gone more deeply into my ideas than necessary, and perhaps not deeply enough into others. I look forward to viewing my classmates projects and comparing them to my own as I feel as though this assignment had a lot of room for variation and I am excited to see how others dealt with this!

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